Proceedings of the American Mathematical Society 2017 vol.145 N7, pages 2859-2869

Finiteness of the number of coideal subalgebras

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Abstract

 It is proved that any finite dimensional Hopf algebra which is either semisimple or cosemisimple has finitely many right coideal subalgebras. As a consequence, over an algebraically closed base field any action of a finite dimensional cosemisimple Hopf algebra on a commutative domain factors through an action of a group algebra. This extends two results of Etingof and Walton to the case where the Hopf algebra is cosemisimple, but not necessarily semisimple.

http://dx.doi.org/10.1090/proc/13463

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